Unlocking the Mysteries of Equations: Writing and Solving 1 and 2 Step Equations for All Students

Math, Differentiated Instruction – Grades 5-8

Students are expected to be able to write and solve one- and two-step equations earlier than ever before. Translating phrases and problems into algebraic expressions and equations, and then solving these equations presents unparalleled difficulty for many students. Learn how to apply the best practices and teaching strategies that will allow students to meet required state and district standards as well as develop meaning that will remain after the test is over. Explore how manipulatives, models, graphic organizers and reading/writing strategies assist in meeting the needs of all students, including limited English proficient learners.

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Connie Colton is a National Board certified teacher teacher. She has spent the past 16 years teaching mathematics for the Omaha Public School District in Omaha, Nebraska. Currently, Connie is the math department chairwoman at McMillan Magnet Center. She serves as a new teacher mentor, a magnet school representative, the math competition coach and an EXCELS School Improvement team leader. Connie provides "Best Practices Workshops" monthly for new mathematics teachers. She was formerly a member of her school district's Criterion Referenced Test writing committee and currently writes curriculum for the district. Connie has been trained in the Carnegie Algebra Tutorial Program and is Banneker Community of Excellence in Mathematics and Science (CEMS) Certified. She holds a bachelor's degree in secondary education with a field endorsement in mathematics from the University of Nebraska at Omaha. Connie is currently pursuing a master's degree in secondary education with an emphasis in mathematics and curriculum development.

Variable Expressions

Notes	Examples
variable a symbol that stands for a number	**examples:** a, b, c, x, y
variable expression a group of numbers, variables, and operations	**examples:** $a + 5$ $\qquad b - c \qquad$ 8c $\quad$ xy $\quad \dfrac{x}{2}$
evaluating variable expressions replacing a variable with a number and finding an answer	**examples:** evaluate for a = 2, b = 3, c = 6 1) $\quad b + c = 3 + 6 = 9$ 2) $\quad b - a = 3 - 2 = 1$ 3) $\quad abc = 2 \bullet 3 \bullet 6 = 36$ 4) $\quad \dfrac{c}{a} = \dfrac{6}{2} = 3$
coefficient the number in front of the variable	**examples:** 2 is the coefficient of 2a 1 is the coefficient of x -1 is the coefficient of -x

Summary:

One-Step Equation Notes	
Notes	**Examples**
Addition: I see +, I do −	ex: $n + 4 = 10$ $\underline{\quad -4 \quad -4}$ $n \quad = \quad 6$ ck: $6 + 4 = 10$
Subtraction: I see − , I do +	ex: $n - 7 = 8$ $\underline{\quad +7 \ +7}$ $n \quad = 15$ ck: $15 - 7 = 8$
Multiplication: I see x , I do ÷	ex: $\dfrac{3n}{3} = \dfrac{12}{3}$ $n = 4$ ck: $3 \cdot 4 = 12$
Division: I see ÷ , I do x	ex: $\dfrac{2}{1} \cdot \dfrac{n}{2} = 9 \cdot 2$ $n = 18$ ck: $\dfrac{18}{2} = 9$
• if written backwards – rewrite to solve the equation if it makes it easier for you • ÷ means to x by the fraction	• -3 + n = 6 rewritten will be $n + -3 = 6$ which can be written $n - 3 = 6$ • $\dfrac{n}{2}$ can be written $\dfrac{1}{2} n$ $\dfrac{n}{5}$ can be written $\dfrac{1}{5} n$
Summary: always do the inverse or opposite operation to solve an equation	

Two-Step Equation Notes	
Notes	**Examples**
Multiplication and Addition: I see x and +, I do ─ and ÷	ex: $2n + 3 = 11$ $\underline{\quad -3 \quad -3}$ $\dfrac{2n}{2} = \dfrac{8}{2}$ $n = 4$ ck: $2 \cdot 4 + 3 = 11$
Multiplication and Subtraction: I see x and ─ , I do + and ÷	ex: $3n - 4 = 14$ $\underline{\quad +4 \quad +4}$ $\dfrac{3n}{3} = \dfrac{18}{3}$ $n = 6$ ck: $3 \cdot 6 - 4 = 14$
Division and Addition: I see ÷ and + , I do ─ and x	ex: $\dfrac{n}{3} + 3 = 7$ $\underline{\quad -3 \quad -3}$ $3 \cdot \dfrac{n}{3} = 4 \cdot 3$ $n = 12$ ck: $\dfrac{12}{3} + 3 = 7$
Division and Subtraction: I see ÷ and ─, I do + and x	ex: $\dfrac{2}{1} \cdot \dfrac{x}{2} = 9 \cdot 2$ $x = 18$ ck: $\dfrac{18}{2} = 9$
Summary:	

Variable Expressions and Equations Lessons

Anticipatory Set:
Have students fold a piece of paper into 4 quadrants. Label each quadrant with the operations: add, subtract, multiply, and divide. Provide students with an overhead warm-up that lists words such as sum, difference, product, quotient, more than, less than, increased by, decreased by, doubled, split, divided equally, shared, totaled, combined, ... Ask students to write the given words under the appropriate operation as a tool that may be used for today's activity.

Objective:
Students will recognize variable expressions for word phrases.
Students will recognize and solve equations from word problems.

Materials:
1. Vocabulary list for the overhead warm-up
2. Student sets of matching word phrases and variable expressions cards (see below)
3. Student sets of matching word problems and equations cards (see below)

Procedure:
- After reviewing the warm-up, have students work in pairs
- Give each pair of students the first set of cards (phrases and expressions)
- Students shuffle the cards and spread them out face down on the table
- Students follow the rules of the game Memory (alternating turns looking for matches)
- When all matches have been found, the teacher should check for accuracy
- Students exchange the phrases and expressions cards for the problems and equations cards
- Students "play" Memory with the word problem and equation cards
- When all matches have been found, the teacher again checks for accuracy
- Finally students should cooperate to solve the equations
- Teacher may ask students to submit solutions in written form, or simply check students' answers at the table

Summary:
Ask students to share with the class the expressions that were the most difficult. Remind students that 'less' and 'less than' as well as 'more' and 'more than' are the most frequently missed, and therefore require the most practice.

Matching word phrases and variable expressions:

1. n + 5   the sum of a number n and 5
2. n − 5   the difference of a number n and 5
3. n • 5   the product of a number n and 5
4. n ÷ 5   the quotient of a number n and 5
5. n + 5   a number n increased by 5
6. n − 5   a number n decreased by 5
7. n • 5   a number n multiplied by 5
8. n ÷ 5   a number n divided by 5
9. n + 5   5 more than a number n
10. n − 5   5 less than a number n
11. n • 5   a number n multiplied by 5
12. n ÷ 5   a number n divided by 5
13. 5 + n   n more than a number 5
14. 5 − n   n less than a number 5
15. 5 • n   a number 5 multiplied by n
16. 5 ÷ n   a number 5 divided by n
17. n + 5   Maria worked 5 more hours than her sister
18. n − 5   Joseph ran 5 miles less than Marcus
19. 5 • n   Eddie earned $5 per hour mowing lawns
20. 5 ÷ n   the class shared 5 pizzas

n + 5	the sum of a number n and 5
n − 5	the difference of a number n and 5
n ● 5	the product of a number n and 5
n ÷ 5	the quotient of a number n and 5

n + 5	a number n increased by 5
n – 5	a number n decreased by 5
n • 5	a number n multiplied by 5
n ÷ 5	a number n divided by 5

n + 5	5 more than a number n
n − 5	5 less than a number n
n • 5	a number n multiplied by 5
n ÷ 5	a number n divided by 5

n + 5	Maria worked 5 more hours than her sister
n – 5	Joseph ran 5 miles less than Marcus
n • 5	Eddie earned $5 per hour mowing lawns
n ÷ 5	the class shared 5 pizzas

Matching word problems and equations:

1. $2n - 40 = 520$ A 2-bedroom apartment that rents for $520 a month is $40 less than twice the cost of a 1-bedroom apartment. What is the monthly rent of a 1-bedroom apartment?

2. $40 \bullet 2 \bullet n \geq 520$ Frances wants to buy a television that costs $520. If she earns $40 every day she works and is able to work 2 days per week, how many weeks will it take her to earn enough money to purchase the TV?

3. $2n + 40 = 520$ Anna earned $520 last summer from babysitting. This amount is $40 more than double the amount she earned over the school year. How much money did she earn during the school year?

4. $40n + 2 = 520$ Robert spent $520 on CDs over the last 4 years. This amount is 2 more than 40 times the average cost of 1 CD. Find the average cost of each CD.

5. $80 - (4 \bullet 12 + 2 \bullet 8) = n$ Melitta bought 4 t-shirts at $12 each and 2 bracelets at $8 each. How much change did she receive from $80?

6. $80 + 4 \bullet 12 - 2 \bullet 8 = n$ Carlos had $80 in his savings account last month. If this month he made 4 deposits of $12 each and 2 withdrawals of $8 each, how much money does he now have in his account?

7. $80 - (4 \bullet 12 + 2 \bullet 8) = n \div 2$ Michael earned 12 points on each of the first 4 questions of his math test, 8 points on each of the next 2 questions, and earned no points on each of the last 2 questions. The total value of the test was 80 points. How many points was each of the last 2 questions worth?

Matching word problems and equations: (cont.)

8. $80 + 4 \bullet 12 + 2 \bullet 8 = n$ — Last week the class collected $80 in donations for the American Heart Association. This week 12 students donated $4 each and 8 students donated $2 each. How much money has the class now collected?

9. $3/4 \bullet 12 = n$ — 3/4 of the students who earned an A on the math test studied for at least 1 hour the night before. If 12 students earned an A, how many students studied for at least 1 hour?

10. $3/4 \bullet n = 12$ — 3/4 of the cookies brought to the bake sale have been sold. If 12 dozen cookies were sold, how many dozen cookies were brought to the bake sale?

11. $12 \div 3/4 = n$ — 12 yards of fabric was to be shared by the sewing class. If each student needed 3/4 of a yard to make a gym-bag, how many students would be able to make a gym-bag?

12. $3/4 \div 12 = n$ — If 3/4 of an hour was given to answer 12 questions, how much time in hours could be spent on each question?

$2n - 40 = 520$	A 2-bedroom apartment that rents for $520 a month is $40 less than twice the cost of a 1-bedroom apartment. What is the monthly rent of a 1-bedroom apartment?
$40 \bullet 2 \bullet n \geq 520$	Frances wants to buy a television that costs $520. If she earns $40 every day she works and is able to work 2 days per week, how many weeks will it take her to earn enough money to purchase the TV?
$2n + 40 = 520$	Anna earned $520 last summer from babysitting. This amount is $40 more than double the amount she earned over the school year. How much money did she earn during the school year?
$40n + 2 = 520$	Robert spent $520 on CDs over the last 4 years. This amount is 2 more than 40 times the average cost of 1 CD. Find the average cost of each CD.
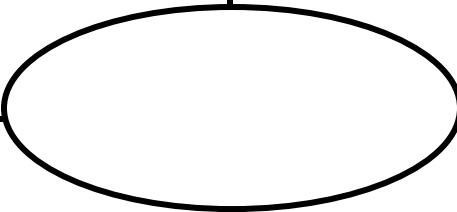
$80 - (4 \bullet 12 + 2 \bullet 8) = n$	Melitta bought 4 t-shirts at $12 each and 2 bracelets at $8 each. How much change did she receive from $80?
$80 + 4 \bullet 12 - 2 \bullet 8 = n$	Carlos had $80 in his savings account last month. If this month he made 4 deposits of $12 each and 2 withdrawals of $8 each, how much money does he now have in his account?
$80 - (4 \bullet 12 + 2 \bullet 8) = n \div 2$	Michael earned 12 points on each of the first 4 questions of his math test, 8 points on each of the next 2 questions, and earned no points on each of the last 2 questions. The total value of the test was 80 points. How many points was each of the last 2 questions worth?
$80 + 4 \bullet 12 + 2 \bullet 8 = n$	Last week the class collected $80 in donations for the American Heart Association. This week 12 students donated $4 each and 8 students donated $2 each. How much money has the class now collected?

$3/4 \bullet 12 = n$	3/4 of the students who earned an A on the math test studied for at least 1 hour the night before. If 12 students earned an A, how many students studied for at least 1 hour?
$3/4 \bullet n = 12$	3/4 of the cookies brought to the bake sale have been sold. If 12 dozen cookies were sold, how many dozen cookies were brought to the bake sale?
$12 \div 3/4 = n$	12 yards of fabric was to be shared by the sewing class. If each student needed 3/4 of a yard to make a gym-bag, how many students would be able to make a gym-bag?
$3/4 \div 12 = n$	If 3/4 of an hour was given to answer 12 questions, how much time in hours could be spent on each question?

Combination Notes

Notes	Graphic Representations

Summary:

Definition

Characteristics

Examples

Non-Examples

+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—

X	X
X	X
-X	-X

$\dfrac{X}{2}$		$\dfrac{X}{2}$		$\dfrac{X}{3}$	$\dfrac{X}{3}$		$\dfrac{X}{3}$
$\dfrac{X}{4}$	$\dfrac{X}{4}$	$\dfrac{X}{4}$	$\dfrac{X}{4}$	$\dfrac{X}{5}$	$\dfrac{X}{5}$	$\dfrac{X}{5}$	$\dfrac{X}{5}$	$\dfrac{X}{5}$